7-4-2016

Enhanced Acoustic Transmission into Dissipative Solid Materials through the Use of Inhomogeneous Plane Waves

Daniel C. Woods
Purdue University, woods41@purdue.edu

J Stuart Bolton
Purdue University, bolton@purdue.edu

Jeffrey F. Rhoads
Purdue University, jfrhoads@purdue.edu

Enhanced Acoustic Transmission into Dissipative Solid Materials through the Use of Inhomogeneous Plane Waves

Daniel C. Woods, J. Stuart Bolton,
and Jeffrey F. Rhoads

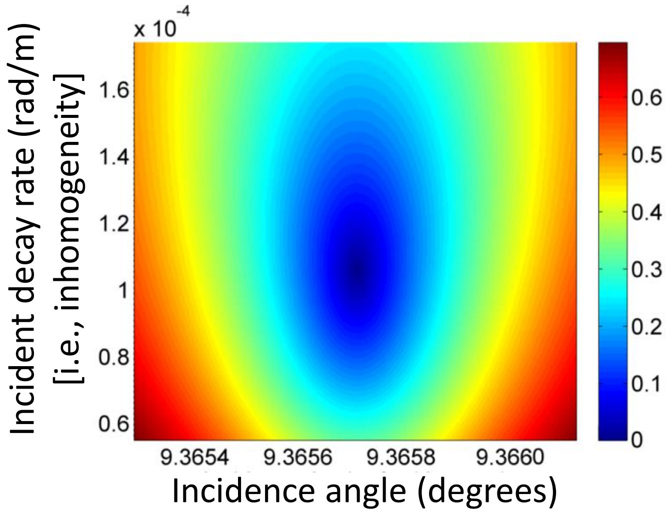
School of Mechanical Engineering,
Ray W. Herrick Laboratories,
and Birck Nanotechnology Center,
Purdue University
West Lafayette, Indiana, USA

July 4, 2016

Introduction

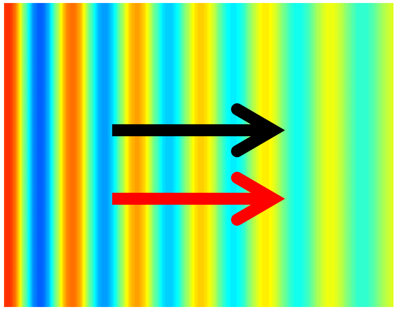
- Study of inhomogeneous plane waves
- Seek to tune the incident wave parameters to maximize energy transmission into dissipative solids

Exemplary Lossless Fluid—Solid Interface: Magnitude of Reflection Coefficient

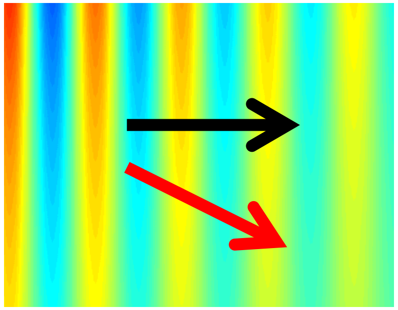


(*adapted from Woods et al.,* 2015)

Homogeneous plane wave



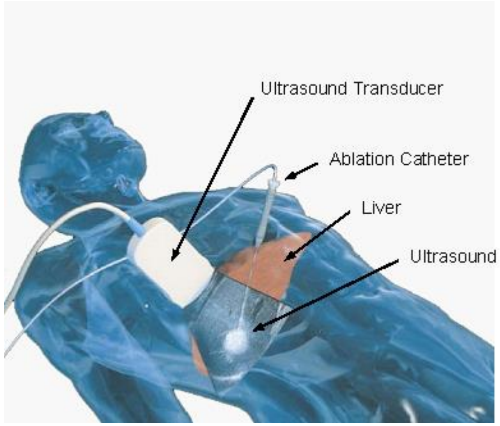
Inhomogeneous plane wave



Introduction

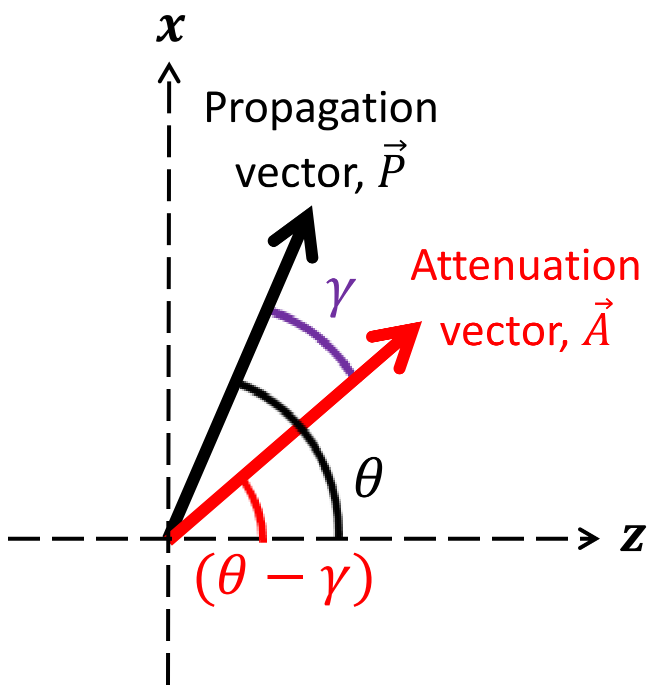
- Potential applications for enhanced acoustic energy transmission into solid materials:
  - Nondestructive structural testing
  - Medical ultrasound imaging and ablation
  - Other, non-contact ultrasound applications

(*Image credit: http://ndtoverseas.org/Conventional%20Non%20Destructive%20Test.html*)



(*Image credit: http://academicdepartments.musc.edu/sebin/x/k/tumor_abl_overview.jpg*)

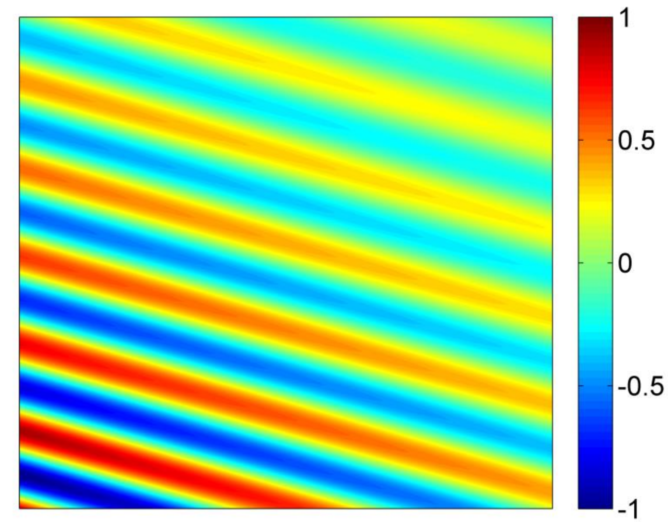
Plane Wave Representation in Dissipative Media



Degree of inhomogeneity,
$0° \leq \gamma < 90°$

Complex wavevector, $\vec{\tilde{\boldsymbol{K}}} = \vec{\boldsymbol{P}} - j\vec{\boldsymbol{A}}$

Corresponding Stress Field (normalized)



Wavevector components:

$$\tilde{k}_x = |\vec{P}|\sin(\theta) - j|\vec{A}|\sin(\theta - \gamma)$$
$$\tilde{k}_z = |\vec{P}|\cos(\theta) - j|\vec{A}|\cos(\theta - \gamma)$$

<u>Further note</u>: γ (inhomogeneity) also affects $|\vec{P}|$ and $|\vec{A}|$ according to the material wavenumber condition $\vec{\tilde{K}} \cdot \vec{\tilde{K}} = \tilde{k}^2$

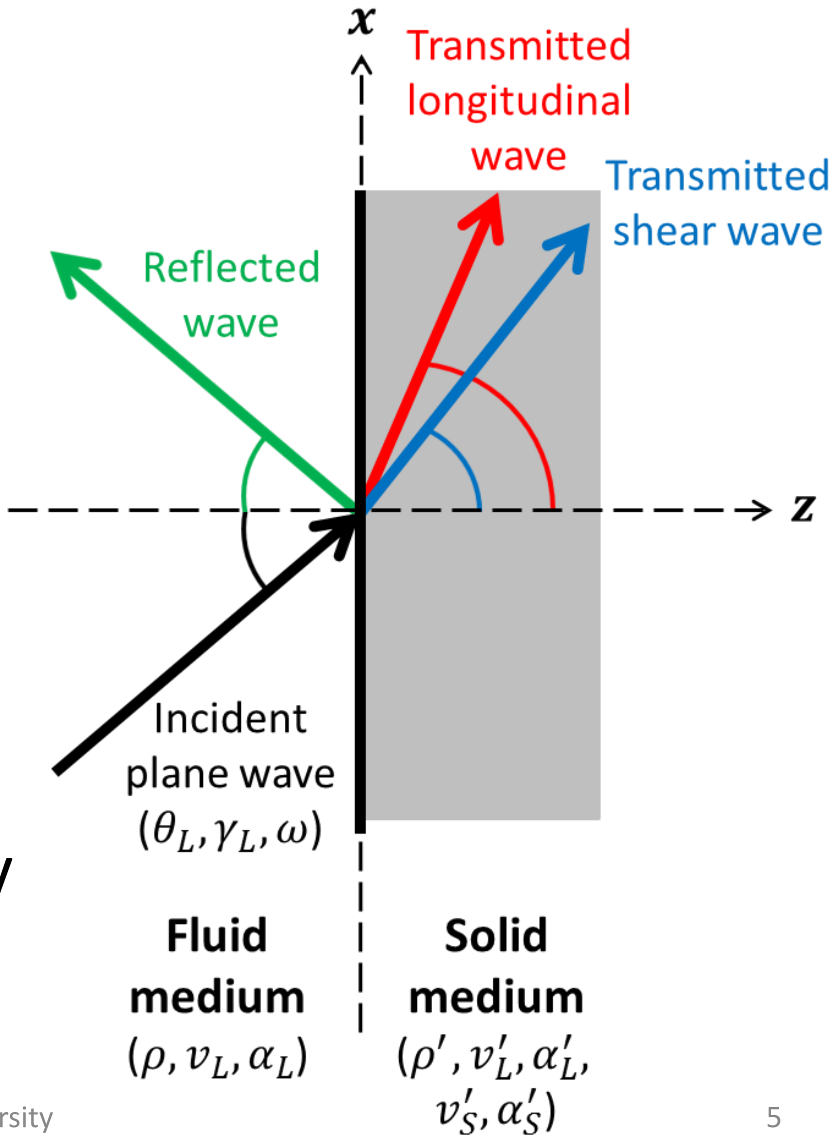
Dissipative Fluid–Solid Interface

- Semi-infinite fluid—solid interface, linear viscoelastic model (hysteretic damping)
- Boundary conditions at interface ($z = 0$)

$$\tilde{\sigma}_{zz}(x,0) = \tilde{\sigma}'_{zz}(x,0)$$
$$\tilde{u}_z(x,0) = \tilde{u}'_z(x,0)$$
$$\tilde{\sigma}'_{xz}(x,0) = 0$$

- Trace wavenumber continuity

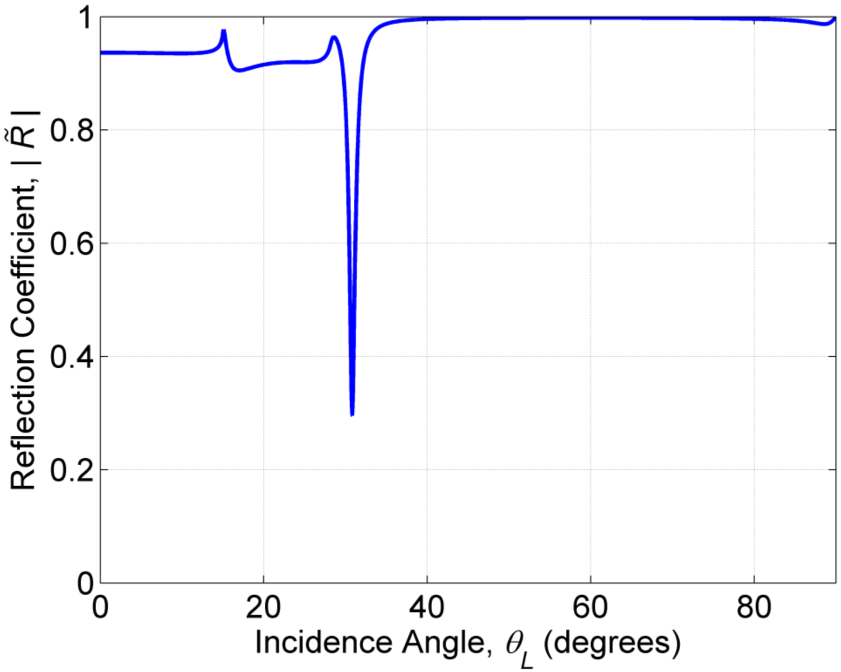
$$\tilde{k}_{L,x} = \tilde{k}'_{L,x} = \tilde{k}'_{S,x}$$



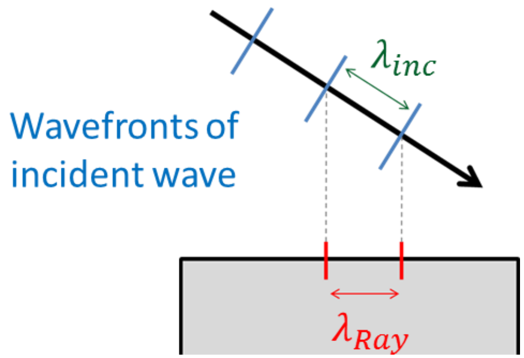
Water—Stainless Steel Interface

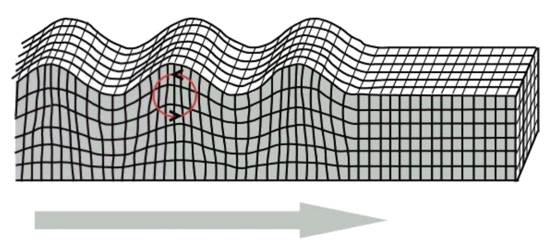
Magnitude of Reflection Coefficient (10 MHz)

Effect of Incidence Angle (Homogeneous Plane Wave)



Reduction of Reflection Coefficient Near Rayleigh Angle





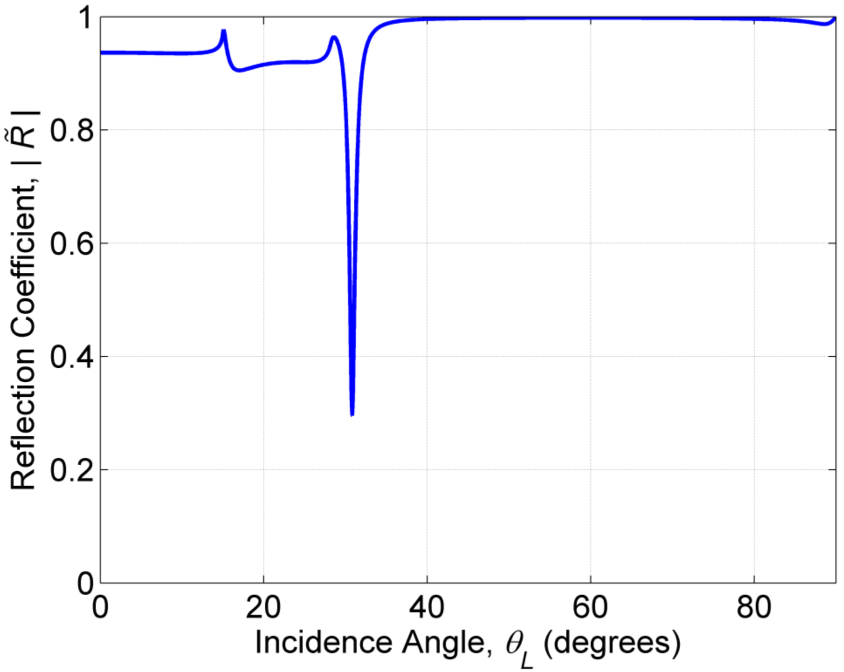
Spatial resonance of induced longitudinal and shear particle motions

(Image credit: http://www.sjvgeology.org/oil/Rayleigh_surface_waves2.gif)

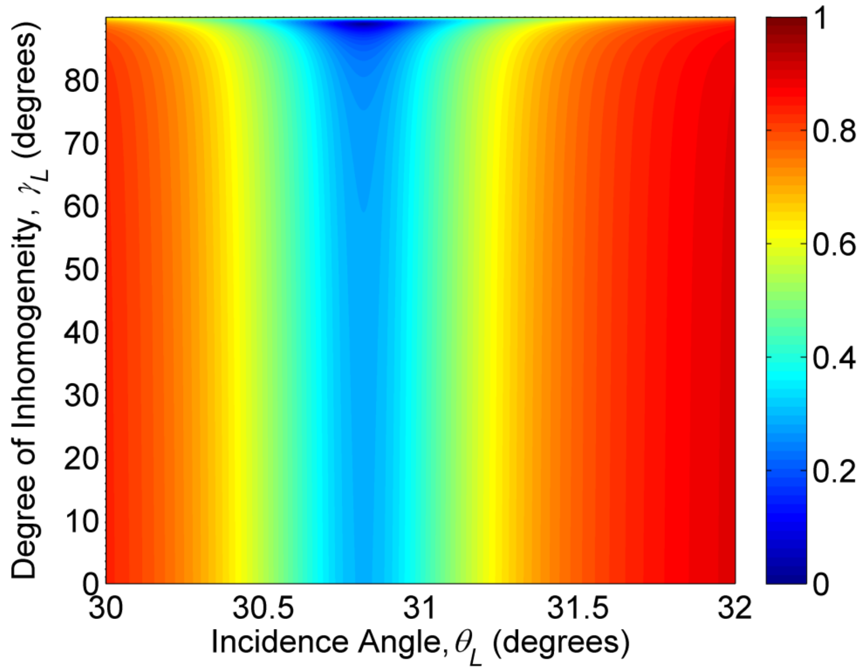
Water—Stainless Steel Interface

Magnitude of Reflection Coefficient (10 MHz)

Effect of Incidence Angle (Homogeneous Plane Wave)



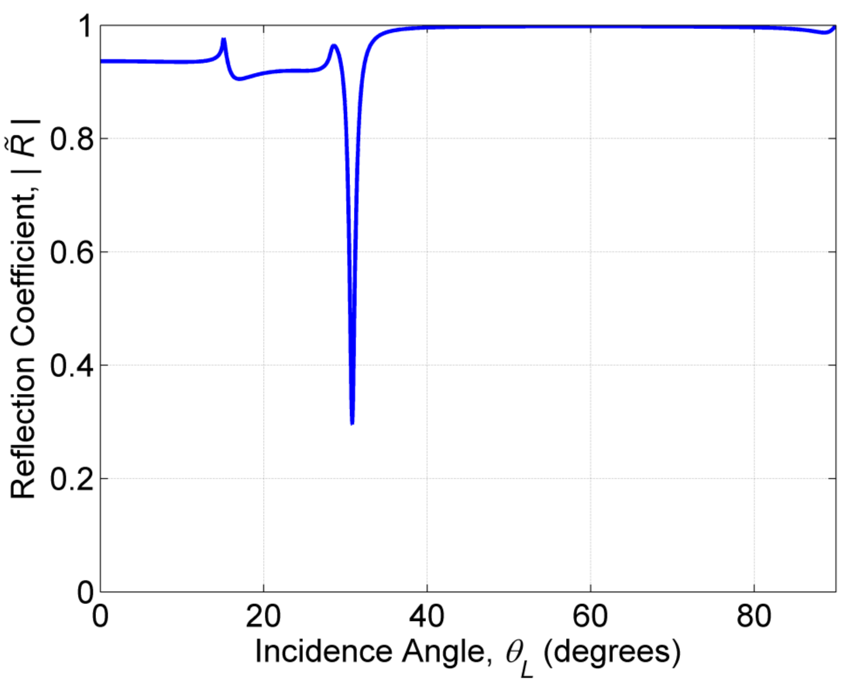
Effect of Wave Inhomogeneity Near Rayleigh Angle



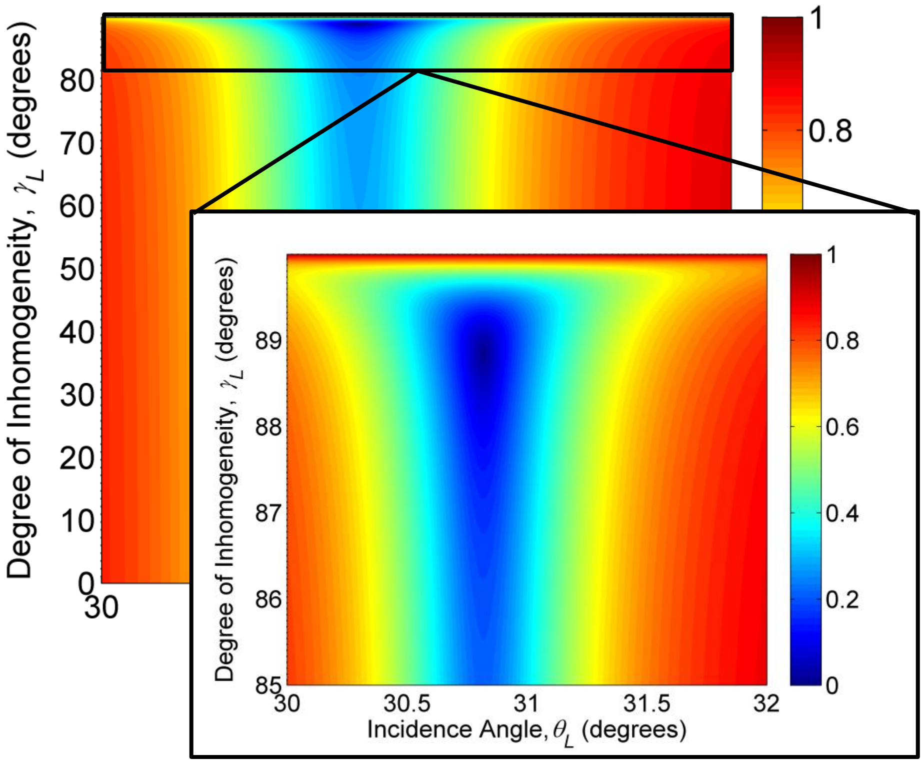
Water—Stainless Steel Interface

Magnitude of Reflection Coefficient (10 MHz)

Effect of Incidence Angle (Homogeneous Plane Wave)



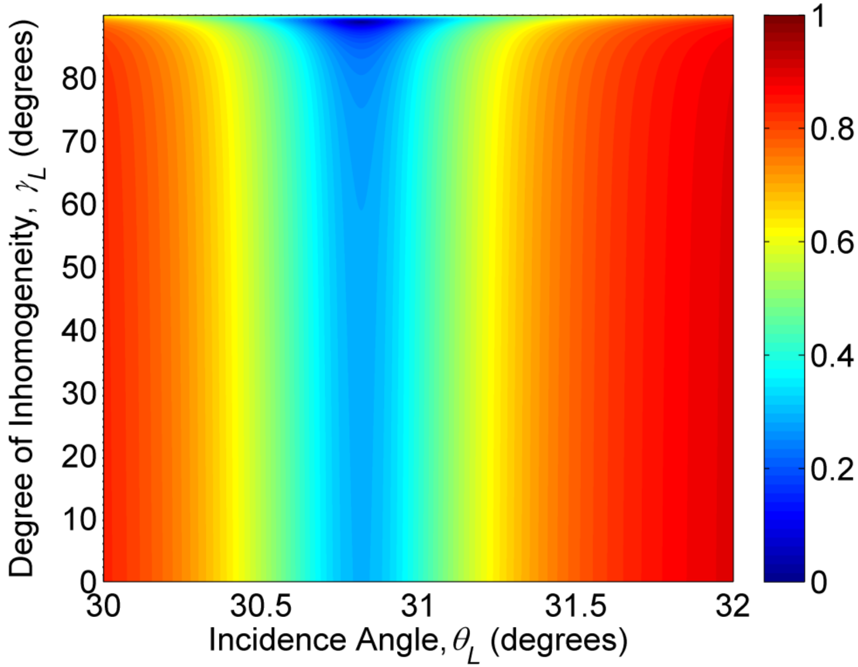
Effect of Wave Inhomogeneity Near Rayleigh Angle



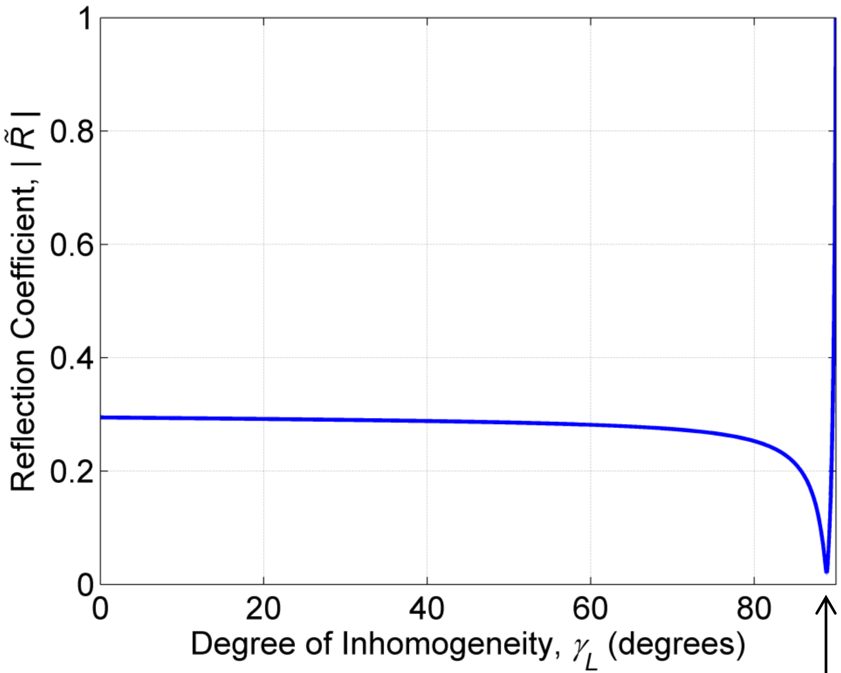
Water—Stainless Steel Interface

Magnitude of Reflection Coefficient (10 MHz)

Effect of Wave Inhomogeneity and Incidence Angle

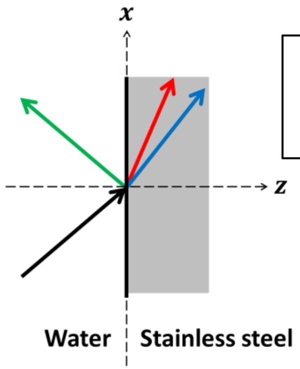


Effect of Wave Inhomogeneity ($\theta_L \approx 30.83°$)



Minimum value at $\gamma_L \approx 88.85°$

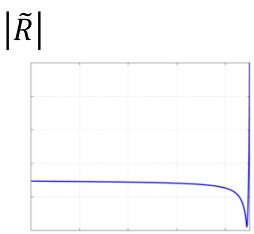
Transmitted Energy Flux



Incident wave amplitude set as 1 Pa at $x = z = 0$

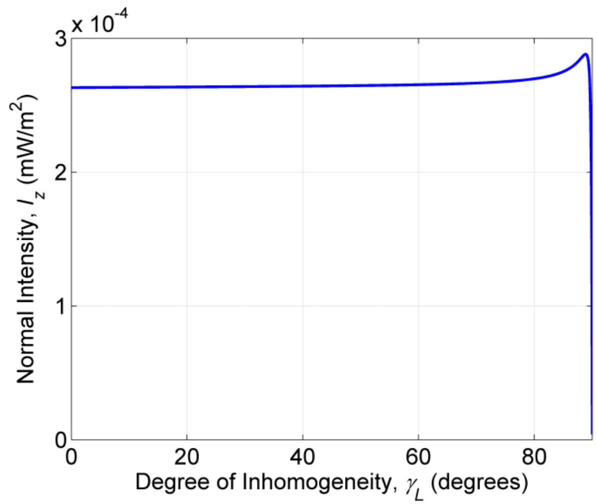
Normal Intensity in Solid

$$I_z = \frac{1}{T}\int_0^T -(\sigma_{zz}v_z + \sigma_{xz}v_x)\,\mathrm{d}t$$

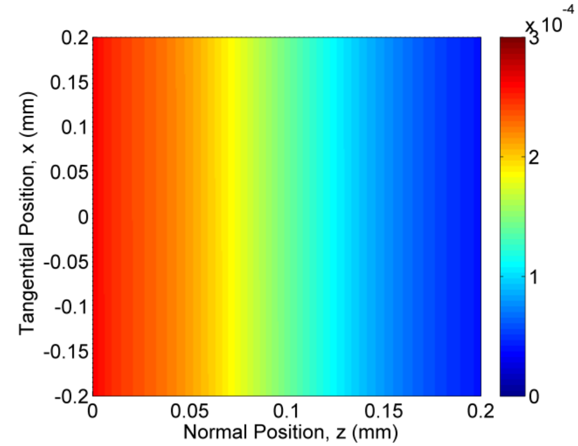


Degree of Inhomogeneity, γ_L

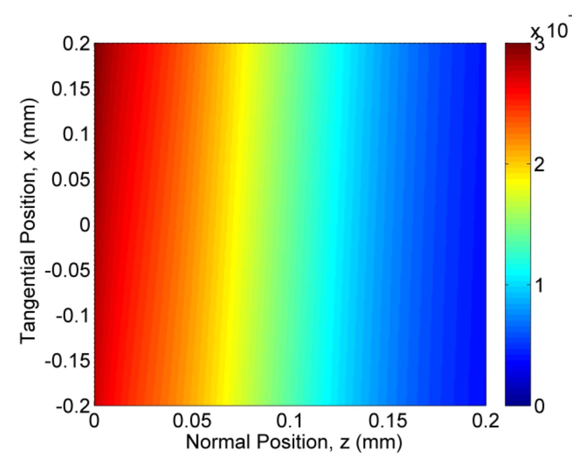
Transmitted Normal Intensity (at the point $x = z = 0$)



Transmitted Normal Intensity Distributions (mW/m³)

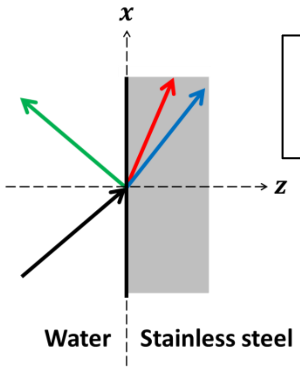


Homogeneous incident wave



Inhomogeneous incident wave

Transmitted Energy Flux

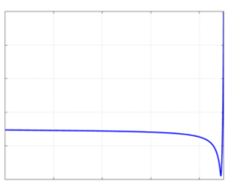


Incident wave amplitude set as 1 Pa at $x = z = 0$

Normal Intensity in Solid

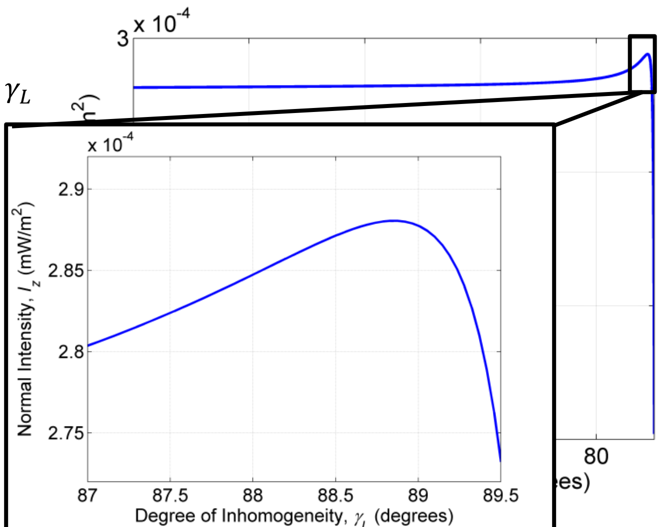
$$I_z = \frac{1}{T}\int_0^T -(\sigma_{zz}v_z + \sigma_{xz}v_x)\,\mathrm{d}t$$

$|\tilde{R}|$

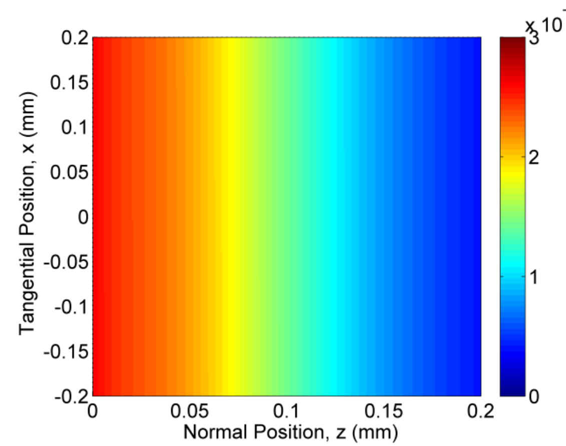


Degree of Inhomogeneity, γ_L

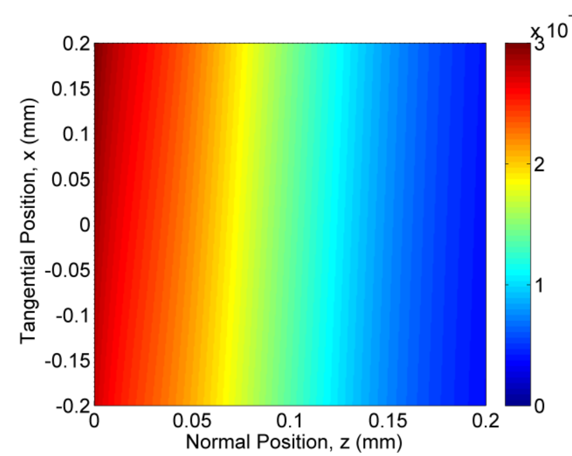
Transmitted Normal Intensity (at the point $x = z = 0$)



Transmitted Normal Intensity Distributions (mW/m³)

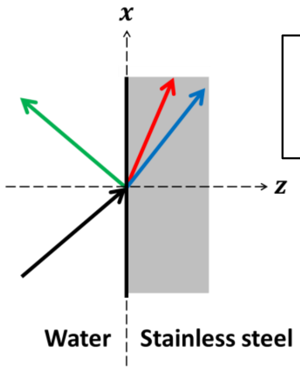


Homogeneous incident wave



Inhomogeneous incident wave

Transmitted Energy Flux

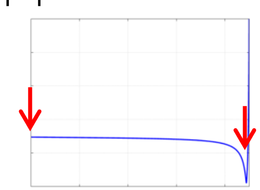


Incident wave amplitude set as 1 Pa at $x = z = 0$

Normal Intensity in Solid

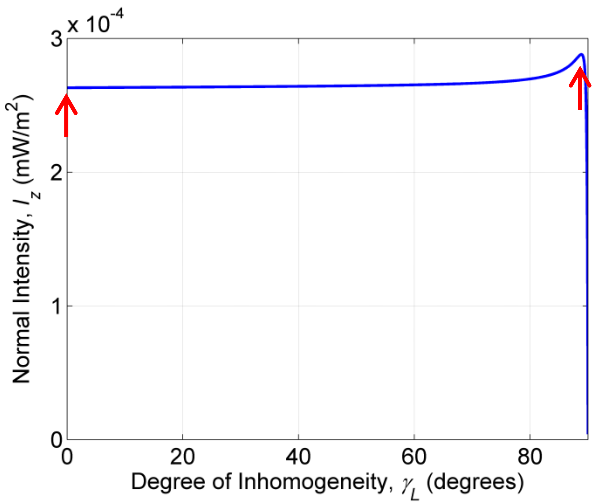
$$I_z = \frac{1}{T}\int_0^T -(\sigma_{zz}v_z + \sigma_{xz}v_x)\,\mathrm{d}t$$

$|\tilde{R}|$

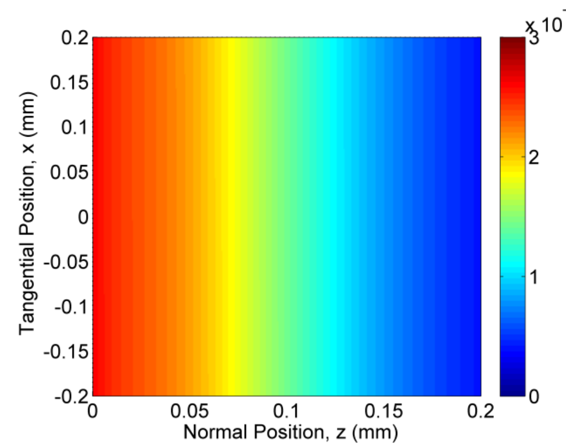


Degree of Inhomogeneity, γ_L

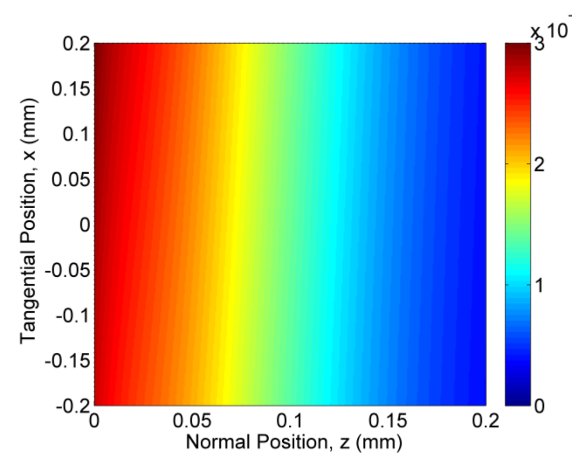
Transmitted Normal Intensity (at the point $x = z = 0$)



Transmitted Normal Intensity Distributions (mW/m³)



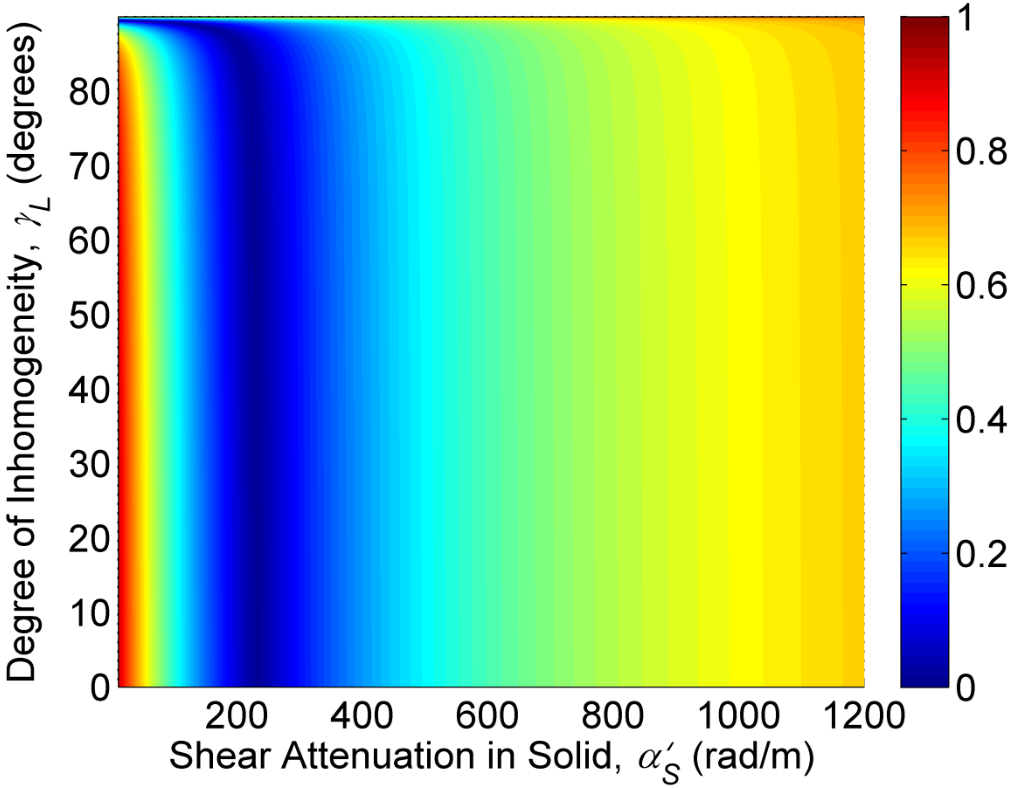
Homogeneous incident wave



Inhomogeneous incident wave

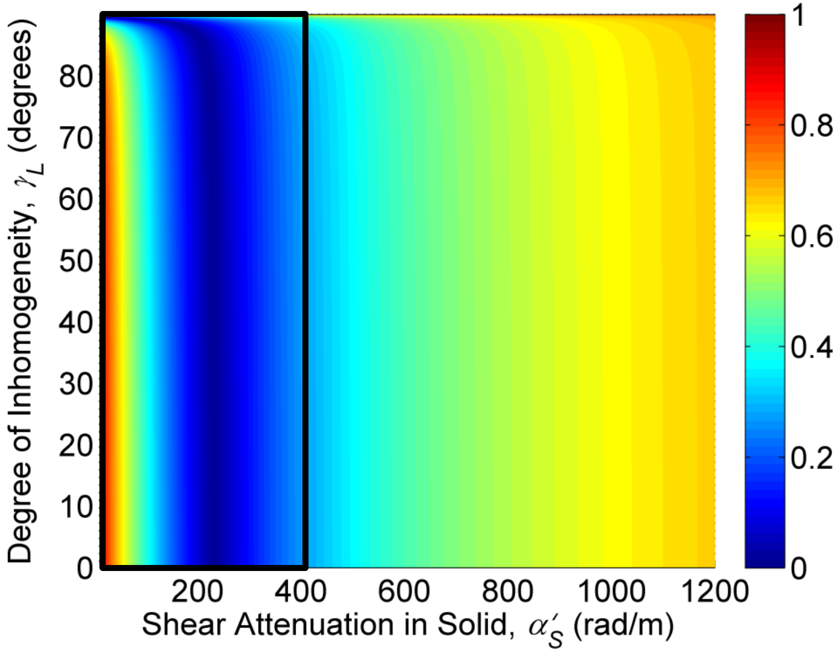
Effect of Increasing Material Dissipation

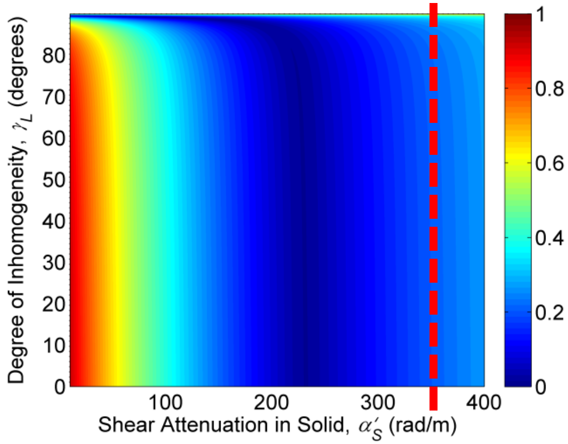
Magnitude of Reflection Coefficient vs. Dissipation Level (10 MHz)

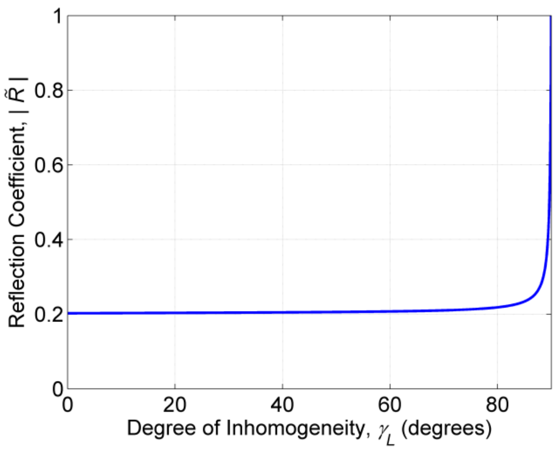


Effect of Increasing Material Dissipation

Magnitude of Reflection Coefficient vs. Dissipation Level (10 MHz)

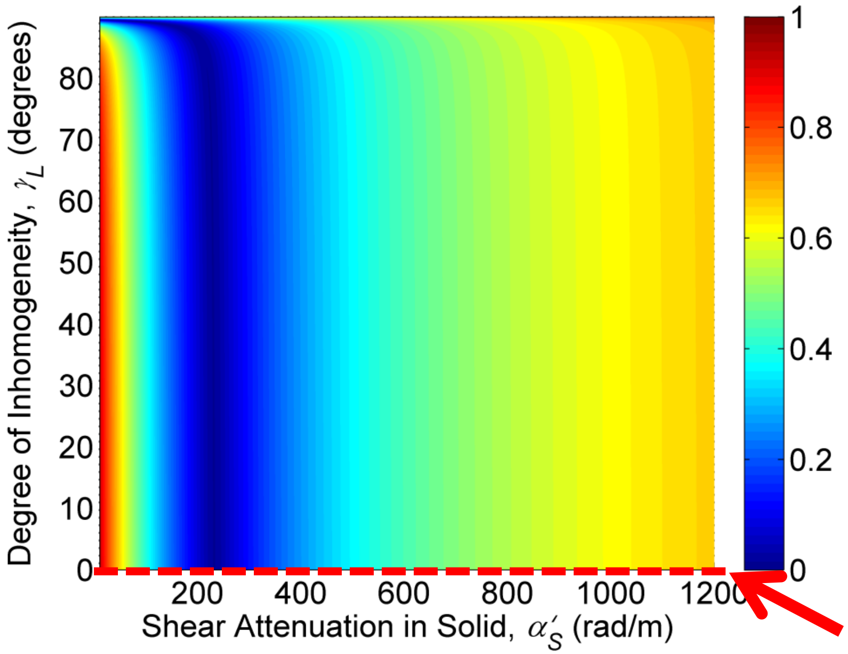


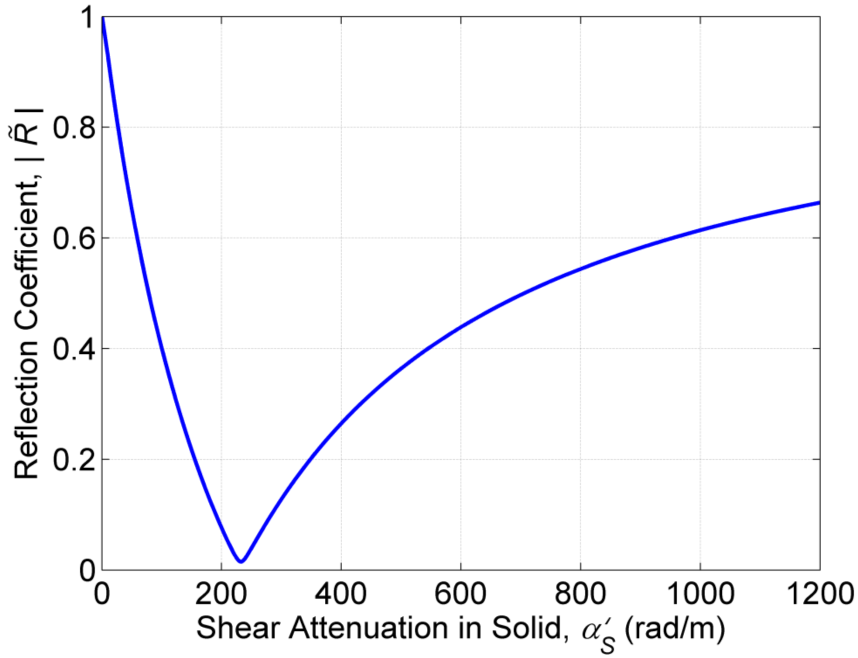




Effect of Increasing Material Dissipation

Magnitude of Reflection Coefficient vs. Dissipation Level (10 MHz)

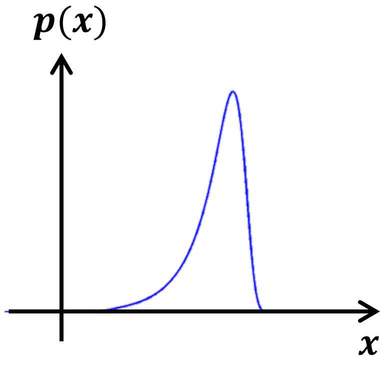




Homogeneous incident wave

Conclusions and Future Work

- Inhomogeneous plane waves incident at dissipative fluid—solid interfaces
  - Significant energy transmission increases for low-loss solids
  - No improvement for higher-loss solids
- Future work:
  - Incorporate frequency-dependence of attenuation
  - Use of bounded wave profiles
  - Generation through phased arrays of sources



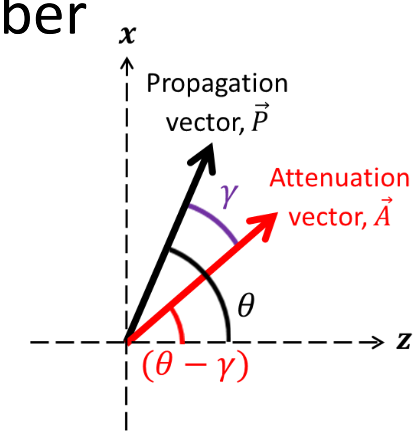
Acknowledgement

The authors would like to thank the U.S. Office of Naval Research for its support of this research under ONR Grant No. N00014-10-1-0958

Plane Wave Representation in Dissipative Media

- Helmholtz equation requires consistency of complex wavevector ($\tilde{\vec{K}} = \vec{P} - j\vec{A}$) with material wavenumber condition:

$$\tilde{\vec{K}} \cdot \tilde{\vec{K}} = \tilde{k}^2 = \left(\frac{\omega}{v_H} - j\alpha_H\right)^2$$



$$\tilde{k}_x = |\vec{P}|\sin(\theta) - j|\vec{A}|\sin(\theta - \gamma)$$
$$\tilde{k}_z = |\vec{P}|\cos(\theta) - j|\vec{A}|\cos(\theta - \gamma)$$

- **It follows that γ (inhomogeneity) affects $|\vec{P}|$ and $|\vec{A}|$:**
  - Inhomogeneous waves ($\gamma \neq 0°$):

$$|\vec{P}|^2 = \frac{1}{2}\left(\mathrm{Re}[\tilde{k}^2] + \sqrt{\left(\mathrm{Re}[\tilde{k}^2]\right)^2 + \frac{\left(\mathrm{Im}[\tilde{k}^2]\right)^2}{\cos^2(\gamma)}}\right) > \left(\frac{\omega}{v_H}\right)^2,$$

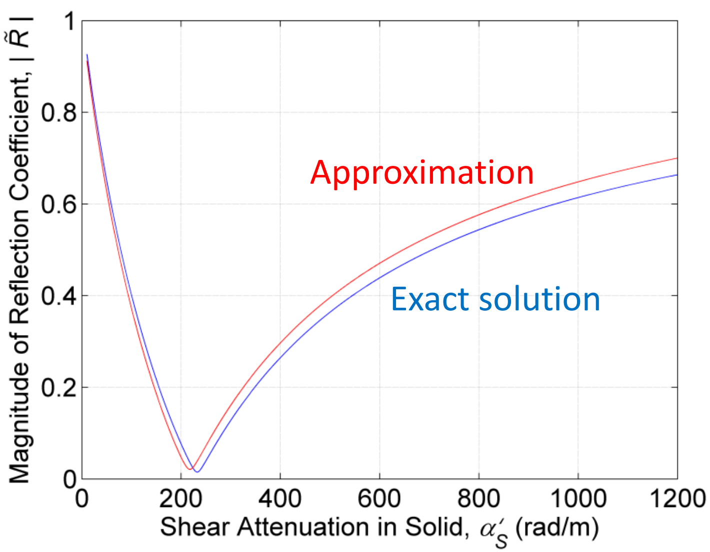
$$|\vec{A}|^2 = \frac{1}{2}\left(-\mathrm{Re}[\tilde{k}^2] + \sqrt{\left(\mathrm{Re}[\tilde{k}^2]\right)^2 + \frac{\left(\mathrm{Im}[\tilde{k}^2]\right)^2}{\cos^2(\gamma)}}\right) > (\alpha_H)^2$$

Approximation for Critical Dissipation Level

- Assumed negligible losses in fluid, small losses in solid

$$\alpha_S'^* \approx \frac{1}{4}\left(\frac{\rho v_L}{\rho' v_S'}\right)\left(\frac{v_{Ray}'}{\sqrt{v_{Ray}'^2 - v_L^2}}\right)\frac{\omega}{V}$$

Magnitude of Reflection Coefficient



Phase of Reflection Coefficient